

# Gribov's Confinement Scenario<sup>1,2</sup>

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## Abstract

I give a brief account of Gribov's scenario of supercritical charges in QCD. Gribov's equation for the Green function of light quarks and its derivation from the corresponding Dyson-Schwinger equation are described. The resulting Green function is shown to exhibit chiral symmetry breaking.

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## 1 Introduction

The confinement of quarks and the breaking of chiral symmetry are among the most striking consequences of Quantum Chromodynamics. In this talk I give a brief account of two interesting suggestions towards their understanding that have been made by V. N. Gribov [1,2]. The first is the idea that confinement is based on the supercritical binding of light quarks. The second is a new approach to the difficult problem of solving the exact Dyson-Schwinger equation for the quark's Green function. This new method has been used to study in detail the breaking of chiral symmetry and the analytic structure of the light quark's Green function [3].

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## 2 Confinement as a supercritical phenomenon

Supercritical charges are well known in QED. An isolated pointlike nucleus with a charge  $Z > 137$  is unstable and captures an electron from the vacuum to form a supercritical bound state while a positron is emitted. This process continues until the charge reaches a subcritical value. The phenomenon of supercritical charges is possible only due to the existence of light fermions making pair creation in the strong field energetically possible. Gribov's idea is that a similar phenomenon causes the confinement of quarks in QCD. In this scenario each color charge is supercritical due to the existence of very light (almost massless) quarks in our world. Contrary to QED this applies even to the color charge of a single quark. The vacuum structure of light quarks is drastically modified due to this process, and the quark becomes a resonance that cannot be observed as an asymptotic state. (For a more detailed discussion the reader is referred to [1,3].) It is expected that the interesting physical picture of supercritical charges manifests itself in the Green function of the light quark to which we now turn.

## 3 Gribov's equation for the Green function of light quarks

In [1] a new equation for the Green function  $G(q)$  of a light quark has been suggested. In order to derive it one starts from the corresponding Dyson–Schwinger integral equation (see for example [4]). It involves the Green function of the gluon which in Feynman gauge is  $D_{\mu\nu}(k) = -g_{\mu\nu}\alpha_s(k)/k^2$ . We assume that the effective strong coupling constant  $\alpha_s(k)$  is a slowly varying function and stays finite at low momenta. This assumption is in agreement with results obtained recently in the dispersive approach to power corrections, see for example [5]. The Feynman gauge is unique in this context because it allows one to obtain an especially simple equation for  $G(q)$ . Its characteristic factor  $1/q^2$  yields a four-dimensional delta-function  $\delta^{(4)}(q)$  under the action of the d'Alembert operator  $\partial^2 = \partial^\mu \partial_\mu$ , where  $\partial_\mu = \partial/\partial q^\mu$ . After applying  $\partial^2$  to the Dyson–Schwinger equation one can identify the most singular contributions – namely the delta-functions – to the integrals from the infrared region where the confining dynamics resides. Collecting only these most singular terms<sup>3</sup> one can perform the integral, and the resulting terms can be shown to involve two factors representing full vertex functions. Using Ward identities the latter can in turn be expressed in terms of derivatives of  $G(q)$  itself. We are then left with a differential equation for the Green function of

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<sup>3</sup> This method can in principle be extended to include also subleading terms.

light quarks in Feynman gauge,

$$\partial^2 G^{-1} = C_F \frac{\alpha_s(q)}{\pi} (\partial^\mu G^{-1}) G (\partial_\mu G^{-1}), \quad (1)$$

where  $C_F = 4/3$ . The running coupling  $\alpha_s(q)$  ensures that the equation reproduces at large spacelike momenta the correct mass and wave function renormalization known from perturbation theory.

## 4 Chiral symmetry breaking and critical coupling

Eq. (1) can be used to study the behaviour of the Green function in different limits analytically. A full numerical study was performed in [3]. We will now restrict ourselves to discussing the dynamical mass function  $M(q^2)$  of the light quark in the region of space-like momenta,  $q^2 < 0$ . It turns out that the Green function shows a characteristically different behaviour if the effective strong coupling constant  $\alpha_s(q)$  exceeds a critical value in some interval of the momentum  $q$  in the infrared region. The critical coupling is found to be

$$\alpha_c = \frac{\pi}{C_F} \left( 1 - \sqrt{\frac{2}{3}} \right) \simeq 0.43. \quad (2)$$

This different behaviour can be shown to correspond to the breaking of chiral symmetry. In order to see this we define a ‘perturbative’ mass  $m_P = M(\lambda^2)$  at some large scale  $\lambda$  where perturbation theory holds. The perturbative mass is, roughly speaking, similar to the current mass of the quark. Let us further define the ‘renormalized’ mass  $m_R = M(0)$  as the low-momentum limit of the dynamical mass function. If the coupling  $\alpha_s$  remains subcritical the relation between  $m_P$  and  $m_R$  is monotonic, and  $m_R$  vanishes for vanishing  $m_P$ . If the coupling becomes larger than  $\alpha_c$  this picture is drastically changed, and the resulting relation between  $m_P$  and  $m_R$  is shown in fig. 1. We see that the renormalized mass remains finite even for vanishing perturbative mass, and chiral symmetry is thus broken.

The breaking of chiral symmetry has been discussed in the context of supercritical charges also in [6], and similar values for the critical coupling have been obtained from different approximations to the Dyson–Schwinger equations, see [4]. A unique feature of the approach outlined above is that eq. (1) is a differential equation which makes it technically very simple to study its solutions in the whole complex momentum plane.

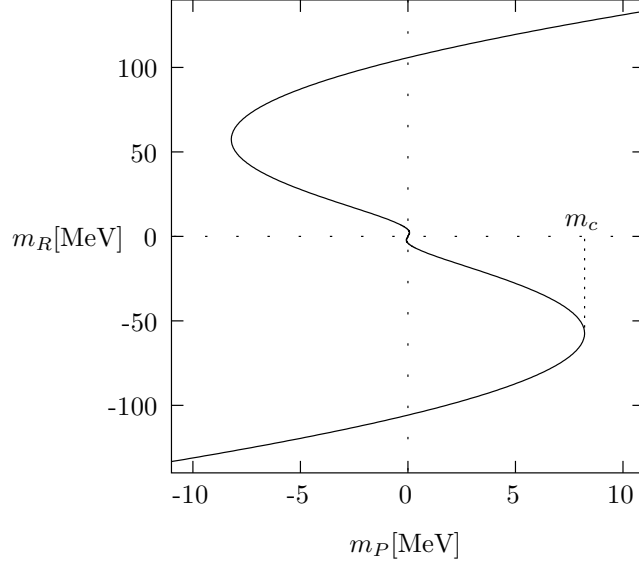


Fig. 1. Relation between perturbative mass  $m_P$  and renormalized mass  $m_R$  (obtained with a model for  $\alpha_s$  motivated by [5])

## 5 Outlook

It turns out that the analytic structure of the Green function obtained from eq. (1) exhibits poles and cuts at positive  $q^2$  and does therefore not correspond to confined quarks [3]. This was expected [1] since this equation does not take into account the effects of pions which arise as Goldstone bosons. The special rôle of pions in the scenario of supercritical charges suggests that they be included as elementary degrees of freedom and that eq. (1) be modified accordingly [2]. There are indications that the modified equation does in fact lead to a confining Green function, but a full analysis still remains to be done.

## References

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